Meeting 2

Math 22

Reading Debrief

- Discuss Tasks 1-3 from Reading 1 w/ the person next to you.
- Are there any questions you want me to address?

Questions to address:

Section 9.1.4 Traces

Definition 9.1.12 A trace of a function $f(x,y)$ in the x direction is a curve defined by the equation

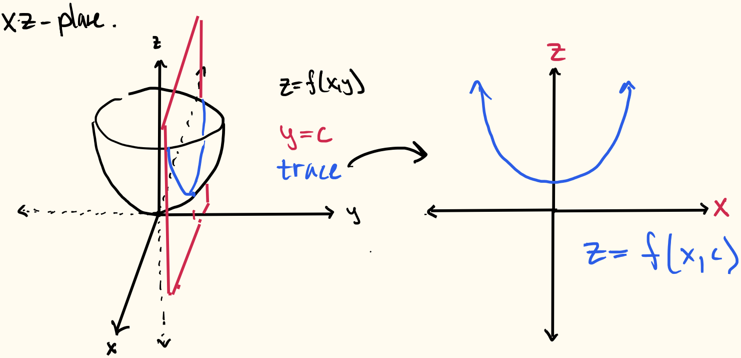
$$z = f(x,c)$$

for some constant $c \in \mathbb{R}$. Similarly, a trace of f in the y direction is a curve of the form

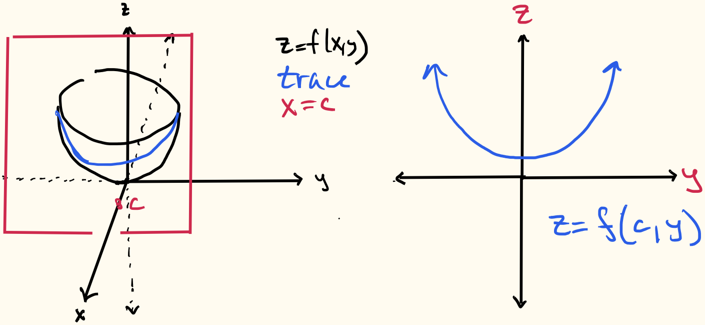
$$z = f(c,y).$$

▨

Traces in the x-direction: intersect the graph of $f(x,y)$ w/ the plane $y=c$. Draw the slice in the xz-plane.



Trace in the y-direction: intersect the graph w/ the plane $x=c$. Draw in yz-plane.



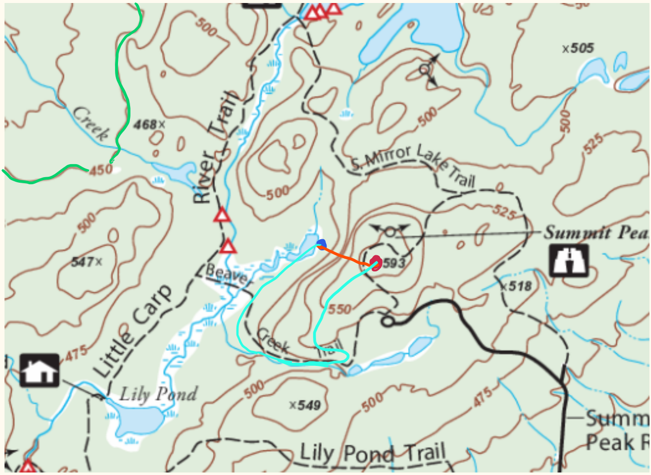
Click the link to see a better visualization

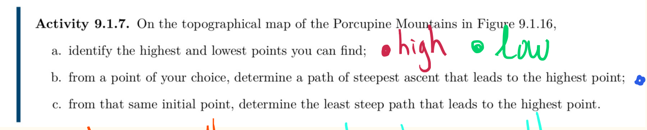
GeoGebra: Traces

Section 9.1.5 Contour Maps and Level Curves

Activity 9.1.7

- Complete as a class.





Activity 9.1.7. On the topographical map of the Porcupine Mountains in Figure 9.1.16,

a. identify the highest and lowest points you can find; • high • low

b. from a point of your choice, determine a path of steepest ascent that leads to the highest point; • chosen pt

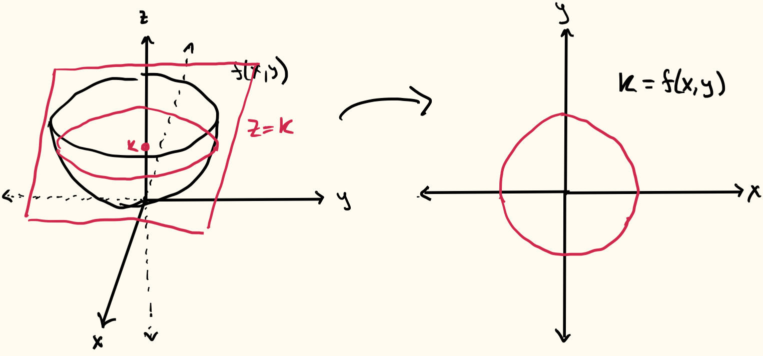
c. from that same initial point, determine the least steep path that leads to the highest point.

• Steep path • not steep path

Definition 9.1.15 A level curve or level contour of a function $f(x,y)$ is a curve of the form $K = f(x,y)$ for some constant

$k \in \mathbb{R}$.

Level curves/contours: similar to traces but no "direction". Intersect graph w/ $z=k$ plane. Plot in xy-plane.



Activity 9.1.8

- Complete Activity 9.1.8 (a) and (c) with the person next to you.
- Skip (b), see Reading 1.2.
- Do not graph the function.
- Class discussion.

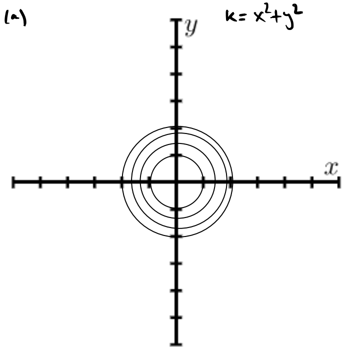
Figure 9.1.19 Left: Level curves for $f(x,y) = x^2 + y^2$. Right: Level curves for $g(x,y) = \sqrt{x^2 + y^2}$.

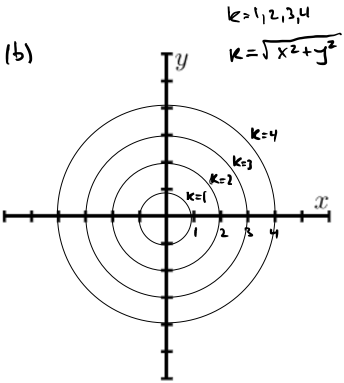
a. Let $f(x,y) = x^2 + y^2$. Draw the level curves $f(x,y) = k$ for $k = 1$, $k = 2$, $k = 3$, and $k = 4$ on the left set of axes given in Figure 9.1.19. (You decide on the scale of the axes.) Explain what the surface defined by f looks like.

b. Let $g(x,y) = \sqrt{x^2 + y^2}$. Draw the level curves $g(x,y) = k$ for $k = 1$, $k = 2$, $k = 3$, and $k = 4$ on the right set of axes given in Figure 9.1.19. (You decide on the scale of the axes.) Explain what the surface defined by g looks like.

c. Compare and contrast the graphs of f and g. How are they alike? How are they different? Use traces for each function to help answer these questions.

Activity 9.1.8.





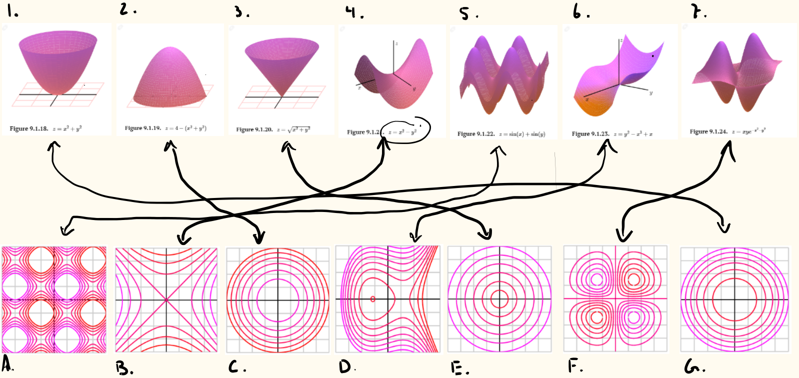
(a) looks like a bowl (parabaloid)

(b) looks like a cone

(c) level curves for (a) and (b) are concentric circles. The radii for (b) grow linearly as a function of k while the radii for (a) do not.

Section 9.1.6 A Gallery of Functions

Match each function with it's level curves:


1.
2.
3.
4.
5.
6.
7.
Figure 9.1.18. $z = x^2 + y^2$
Figure 9.1.19. $z = 4 - (x^2 + y^2)$
Figure 9.1.20. $z = \sqrt{x^2 + y^2}$
Figure 9.1.2 $z = x^2 - y^2$
Figure 9.1.22. $z = \sin(x) + \sin(y)$
Figure 9.1.23. $z = y^2 - x^3 + x$
Figure 9.1.24. $z = xye^{-x^2-y^2}$
x
y
A.
B.
C.
D.
E.
F.
G.

End of Section 9.1.

Section 9.2

Vectors

Reading Assignment Debrief

- Discuss 4-7 from Reading 1 w/ the person next to you.
- Are there any questions from Sections 9.2.1 - 9.2.2 that you want me to address?

Questions: None

Definition 9.2.1 A **vector** is a quantity that possesses the attributes of both magnitude and direction.

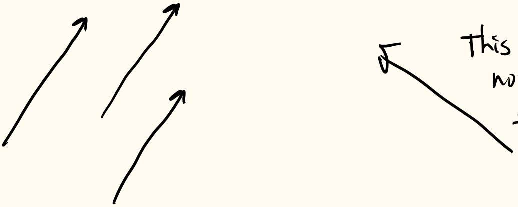
Examples:
- displacement
- acceleration
- force
- velocity

Non-Examples
- speed
- distance
- rate of change of speed

These are all the magnitude of a vector.

Geometric Representation of Vectors

Arrows



This vector is not equal to the others.

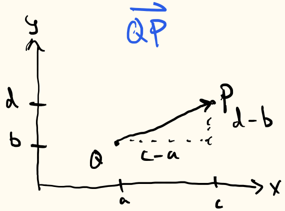
All these vectors are the same since they have the same magnitude and direction.

Displacement Vectors

Let $Q=(a,b)$ and $P=(c,d)$

$\overrightarrow{QP} = \langle c-a, d-b \rangle$

Displacement Vector from Q to P.



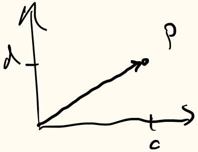
You can do the same thing in 3 or more dim.

Position Vectors

Any point $P=(c,d)$ determines a unique vector by taking the initial point to be the origin $O=(0,0)$

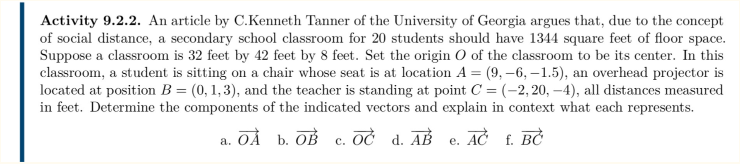
$$\overrightarrow{OP} = \langle c, d \rangle$$

This is the **Position Vector for P**



Activity 9.2.2

- Complete activity and discuss w/ the person next to you.
- Class discussion.



Activity 9.2.2. An article by C.Kenneth Tanner of the University of Georgia argues that, due to the concept of social distance, a secondary school classroom for 20 students should have 1344 square feet of floor space. Suppose a classroom is 32 feet by 42 feet by 8 feet. Set the origin O of the classroom to be its center. In this classroom, a student is sitting on a chair whose seat is at location $A = (9, -6, -1.5)$, an overhead projector is located at position $B = (0, 1, 3)$, and the teacher is standing at point $C = (-2, 20, -4)$, all distances measured in feet. Determine the components of the indicated vectors and explain in context what each represents.

a. $\overrightarrow{OA}$ b. $\overrightarrow{OB}$ c. $\overrightarrow{OC}$ d. $\overrightarrow{AB}$ e. $\overrightarrow{AC}$ f. $\overrightarrow{BC}$

(a) $\overrightarrow{OA} = \langle 9, -6, -1.5 \rangle$ Displacement from the center of the room to the student.

(b) $\overrightarrow{OB} = \langle 0, 1, 3 \rangle$ Replace student w/ projector

(c) $\overrightarrow{OC} = \langle -2, 20, -4 \rangle$ Replace projector w/ teacher.

(d) $\overrightarrow{AB} = B - A = \langle -9, 7, 4.5 \rangle$

Displacement between student and projector.

(e) $\overrightarrow{AC} = C - A = \langle -11, 26, -2.5 \rangle$

Displacement from student to teacher.

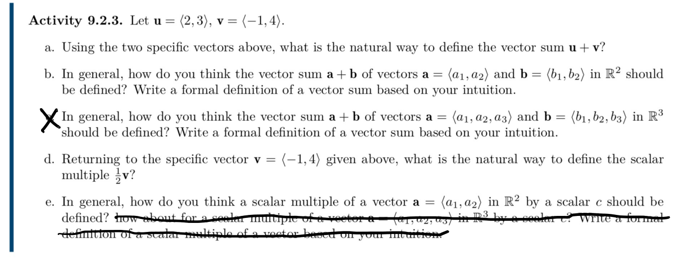
(f) $\overrightarrow{BC} = C - B = \langle -2, 19, -7 \rangle$

Displacement from projector to teacher.

Section 9.2.3 Operations on Vectors

Activity 9.2.3

- Complete the activity and discuss w/ group.
- Class discussion.



Activity 9.2.3. Let $\mathbf{u} = \langle 2,3\rangle$, $\mathbf{v} = \langle -1,4\rangle$.

a. Using the two specific vectors above, what is the natural way to define the vector sum $\mathbf{u}+\mathbf{v}$?

b. In general, how do you think the vector sum $\mathbf{a}+\mathbf{b}$ of vectors $\mathbf{a} = \langle a_1, a_2\rangle$ and $\mathbf{b} = \langle b_1, b_2\rangle$ in $\mathbb{R}^2$ should be defined? Write a formal definition of a vector sum based on your intuition.

X In general, how do you think the vector sum $\mathbf{a}+\mathbf{b}$ of vectors $\mathbf{a} = \langle a_1, a_2, a_3\rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3\rangle$ in $\mathbb{R}^3$ should be defined? Write a formal definition of a vector sum based on your intuition.

d. Returning to the specific vector $\mathbf{v} = \langle -1,4\rangle$ given above, what is the natural way to define the scalar multiple $\frac{1}{2}\mathbf{v}$?

e. In general, how do you think a scalar multiple of a vector $\mathbf{a} = \langle a_1, a_2\rangle$ in $\mathbb{R}^2$ by a scalar c should be defined? ~~How about for a scalar multiple of a vector $\mathbf{a} = \langle a_1, a_2, a_3\rangle$ in $\mathbb{R}^3$ by a scalar c? Write a formal definition of a scalar multiple of a vector based on your intuition.~~

(a) $u+v = \langle 2,3\rangle + \langle -1,4\rangle = \langle 1,7\rangle$

(b) Definition: For all $a,b \in \mathbb{R}^2$,

$$a+b = \langle a_1,a_2\rangle + \langle b_1,b_2\rangle = \langle a_1+b_1,\ a_2+b_2\rangle$$

vector addition

(c) Definition: For all $a,b \in \mathbb{R}^3$,

$$a+b = \langle a_1,a_2,a_3\rangle + \langle b_1,b_2,b_3\rangle = \langle a_1+b_1,\ a_2+b_2,\ a_3+b_3\rangle$$

(d) $\frac{1}{2}v = \langle \frac{1}{2}(-1), \frac{1}{2}4\rangle = \langle -\frac{1}{2}, 2\rangle$

(e) Definition: For all $c \in \mathbb{R}$, $a \in \mathbb{R}^3$,

$$c\cdot a = c\langle a_1,a_2,a_3\rangle = \langle ca_1, ca_2, ca_3\rangle$$

Scalar mult.

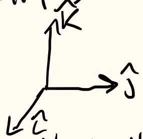
Vector Subtraction Let $u,v \in \mathbb{R}^2$ or $\mathbb{R}^3$

$$v - u := v + (-1)u.$$

Standard Unit Vectors Three special vectors in $\mathbb{R}^3$

$$\hat{i} = \langle 1,0,0 \rangle \quad \hat{j} = \langle 0,1,0 \rangle \quad \hat{k} = \langle 0,0,1 \rangle$$

Every vector $v = \langle a,b,c \rangle$ can be written uniquely as a sum of scalar multiples of $\hat{i}, \hat{j}, \hat{k}$.



$$\langle a,b,c \rangle = a\hat{i} + b\hat{j} + c\hat{k}$$

The vectors $\hat{i}, \hat{j}, \hat{k}$ have length 1 (unit length)

Section 9.2.4 Properties of Vector Operations

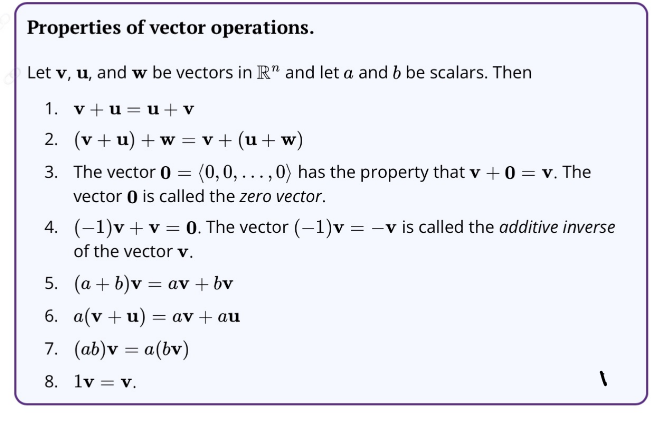
Vector addition and scalar mult have properties similar to addition/mult of real numbers.

Ex $\quad u + v = \langle a,b \rangle + \langle c,d \rangle \quad$ (commutative)

$$= \langle a+c, b+d \rangle$$

$$= \langle c+a, d+b \rangle = v + u$$

Properties of Vector Operations



Properties of vector operations.

Let $\mathbf{v}$, $\mathbf{u}$, and $\mathbf{w}$ be vectors in $\mathbb{R}^n$ and let a and b be scalars. Then

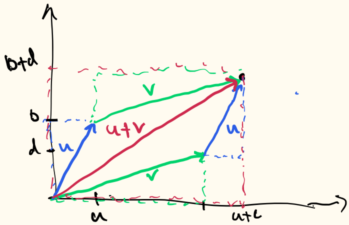
1. $\mathbf{v}+\mathbf{u}=\mathbf{u}+\mathbf{v}$
2. $(\mathbf{v}+\mathbf{u})+\mathbf{w}=\mathbf{v}+(\mathbf{u}+\mathbf{w})$
3. The vector $\mathbf{0}=\langle 0,0,\ldots,0\rangle$ has the property that $\mathbf{v}+\mathbf{0}=\mathbf{v}$. The vector $\mathbf{0}$ is called the *zero vector*.
4. $(-1)\mathbf{v}+\mathbf{v}=\mathbf{0}$. The vector $(-1)\mathbf{v}=-\mathbf{v}$ is called the *additive inverse* of the vector $\mathbf{v}$.
5. $(a+b)\mathbf{v}=a\mathbf{v}+b\mathbf{v}$
6. $a(\mathbf{v}+\mathbf{u})=a\mathbf{v}+a\mathbf{u}$
7. $(ab)\mathbf{v}=a(b\mathbf{v})$
8. $1\mathbf{v}=\mathbf{v}$.

Section 9.2.5 Geometric Interpretation of Vector Operations

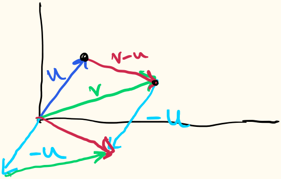
let $u=\langle a,b\rangle$ and $v=\langle c,d\rangle$.

Addition We want to see what happens geometrically when we add two vectors.

$$u+v=\langle a+c,\ b+d\rangle$$

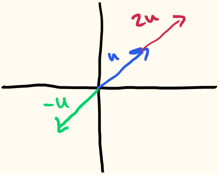


Subtraction $v-u=v+(-1)u$



v-u is the arrow which points from the terminal point of u to the terminal point of v.

Scalar Multiplication



Activity 9.2.4

- Complete Activity 9.2.4 and discuss with your group.
- Class discussion.

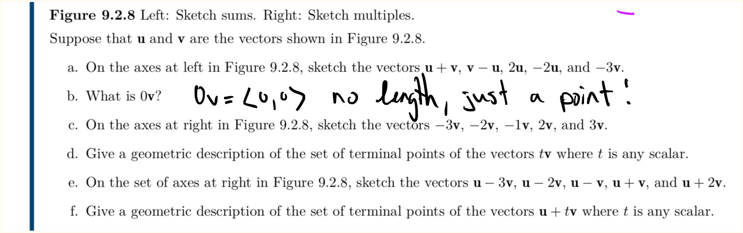


Figure 9.2.8 Left: Sketch sums. Right: Sketch multiples.

Suppose that **u** and **v** are the vectors shown in Figure 9.2.8.

a. On the axes at left in Figure 9.2.8, sketch the vectors $\mathbf{u}+\mathbf{v}$, $\mathbf{v}-\mathbf{u}$, $2\mathbf{u}$, $-2\mathbf{u}$, and $-3\mathbf{v}$.

b. What is $0\mathbf{v}$? $0v = \langle 0,0 \rangle$ no length, just a point!

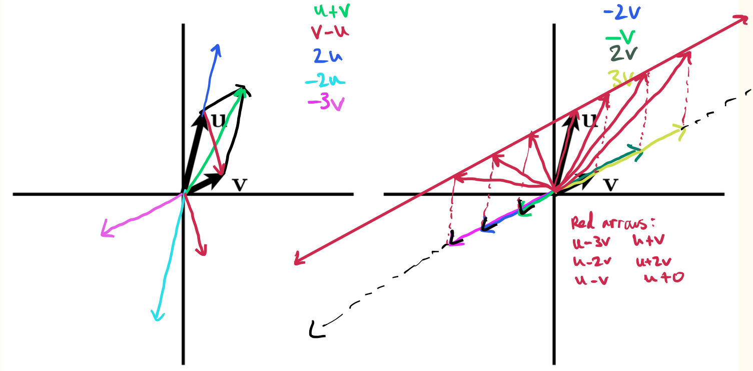
c. On the axes at right in Figure 9.2.8, sketch the vectors $-3\mathbf{v}$, $-2\mathbf{v}$, $-1\mathbf{v}$, $2\mathbf{v}$, and $3\mathbf{v}$.

d. Give a geometric description of the set of terminal points of the vectors $t\mathbf{v}$ where t is any scalar.

e. On the set of axes at right in Figure 9.2.8, sketch the vectors $\mathbf{u}-3\mathbf{v}$, $\mathbf{u}-2\mathbf{v}$, $\mathbf{u}-\mathbf{v}$, $\mathbf{u}+\mathbf{v}$, and $\mathbf{u}+2\mathbf{v}$.

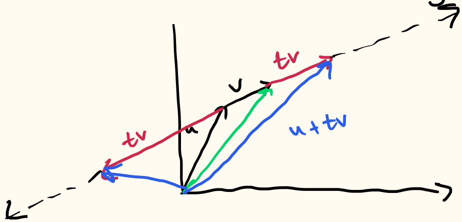
f. Give a geometric description of the set of terminal points of the vectors $\mathbf{u}+t\mathbf{v}$ where t is any scalar.

Activity 9.2.4.



(d) The set $\{tv \mid t \in \mathbb{R}\}$ is a line through the origin in the direction of v.

(e) The set $\{u + tv \mid t \in \mathbb{R}\}$ is the line from (d) translated by u.



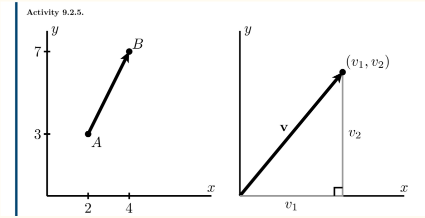
Section 9.2.6 Magnitude

Activity 9.2.5

- Complete and discuss w/ the person next to you.

- Class discussion.

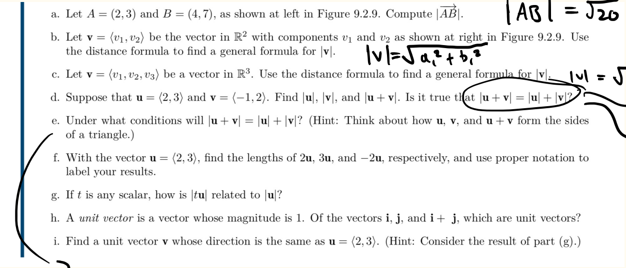
Activity 9.2.5.



Distance formula

$P=(a,b)$ $Q=(c,d)$

$|PQ| = \sqrt{(c-a)^2+(d-b)^2}$



a. Let $A = (2,3)$ and $B = (4,7)$, as shown at left in Figure 9.2.9. Compute $|\overrightarrow{AB}|$.

$|\overrightarrow{AB}| = \sqrt{20}$

b. Let $\mathbf{v} = \langle v_1, v_2 \rangle$ be the vector in $\mathbb{R}^2$ with components v_1 and v_2 as shown at right in Figure 9.2.9. Use the distance formula to find a general formula for $|\mathbf{v}|$.

$|v| = \sqrt{a_1^2 + b_1^2}$

c. Let $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$ be a vector in $\mathbb{R}^3$. Use the distance formula to find a general formula for $|\mathbf{v}|$.

$|v| = \sqrt{a_1^2 + b_1^2 + c_1^2}$

d. Suppose that $\mathbf{u} = \langle 2,3 \rangle$ and $\mathbf{v} = \langle -1,2 \rangle$. Find $|\mathbf{u}|$, $|\mathbf{v}|$, and $|\mathbf{u}+\mathbf{v}|$. Is it true that $|\mathbf{u}+\mathbf{v}| = |\mathbf{u}| + |\mathbf{v}|$?

not true

$|u| = \sqrt{13}$

$|v| = \sqrt{5}$

$|u+v| = \sqrt{26}$

e. Under what conditions will $|\mathbf{u}+\mathbf{v}| = |\mathbf{u}| + |\mathbf{v}|$? (Hint: Think about how $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{u}+\mathbf{v}$ form the sides of a triangle.)

f. With the vector $\mathbf{u} = \langle 2,3 \rangle$, find the lengths of $2\mathbf{u}$, $3\mathbf{u}$, and $-2\mathbf{u}$, respectively, and use proper notation to label your results.

g. If t is any scalar, how is $|t\mathbf{u}|$ related to $|\mathbf{u}|$?

h. A *unit vector* is a vector whose magnitude is 1. Of the vectors $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{i} + \mathbf{j}$, which are unit vectors?

i. Find a unit vector $\mathbf{v}$ whose direction is the same as $\mathbf{u} = \langle 2,3 \rangle$. (Hint: Consider the result of part (g).)

(e) If u and v are parallel, then $|u+v| = |u| + |v|$.

(f) $|2u| = 2|u|$ $|3u| = 3|u|$ $|-2u| = 2|u|$.

(g) $|tu| = |t|\,|u|$

(h) $|\hat{i}| = 1$ $|\hat{j}| = 1$ $|\hat{i}+\hat{j}| = \sqrt{2}$

Sum of unit vectors is not a unit vector

(i) Assume $v = cu = \langle c\cdot 2, c\cdot 3 \rangle$

$1 = |v| = |c|\cdot|u| = |c|\cdot\sqrt{13}$

$c = \pm \frac{1}{\sqrt{13}}$ $v = \pm\frac{1}{\sqrt{13}}\langle 2,3 \rangle$.